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FRISCH ELASTICITY AND FISCAL MULTIPLIERS

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Abstract

The literature on fiscal multipliers has evolved towards the notion that there is no such think as *a* fiscal multiplier. These differ among many dimensions, including the aggregate state of the economy, wealth and income inequality, the demographic structure to name a few. However, in all of this studies, the elasticity of intertemporal substitution (Frisch) is held constant across countries. Based on findings from Peterman (2015), we use micro data to estimate country specific Frisch elasticities and find that these differences not only are quantitatively meaningful in explaining cross-country differences in fiscal multipliers, but that the magnitude of the differences is much larger than in most mechanisms that have been brought forward to explain cross-country heterogeneity in fiscal multipliers.

Keywords: *Frisch elasticity, Fiscal multipliers, Incomplete markets, Fiscal consolidation*

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1 Introduction

In the past decade, the analysis of heterogeneous agents has been one of the most vibrant topics in the macro literature. Alongside a renewed interest on the effects of fiscal policy, much focus has been devoted to understand how fiscal multipliers vary across time and space. In particular, it has been documented in the literature how different economic structural characteristics shape the aggregate responses to fiscal stimulus and account for substantial cross-country variation in fiscal multipliers. Such explorations included supply-side mechanisms that greatly rely on the behaviour of the labor supply. Brinca et al. (2016), for instance, documents how the degree of wealth inequality accounts for different aggregate responses to fiscal stimulus when it implies a higher fraction of credit constrained households that turn aggregate labor supply less responsive to future income shocks. Additionally, Brinca et al. (2019*b*) documents how the labor supply responses to a precautionary savings motive may account for the positive relationship between the recessive effects of a contractionary fiscal policy and income inequality[1]. Hence, in this context, we propose to assess the quantitatitve relevance of accounting for country-specific magnitudes of the intertemporal substitution of the labor supply in the aggregate response to fiscal consolidation.

In parallel, the intertemporal substitution of the labor supply – also referred to as Frisch elasticity – has been at the center of a long-standing debate in the literature. The controversy emerges from contradictory micro and macro-level evidence on the magnitude of this elasticity. Historically, findings from micro-level data presented in MaCurdy (1981) and Altonji (1986) helped to form the view that the Frisch elasticity is small, not surpassing 0.5. However, as discussed by Reichling and Whalen (2012), that view was challenged by macroeconomic literature that found required to use an elasticity ranging from 2 to 4, so as to replicate observed aggregate fluctuations (King and Rebelo (1999) and Smets and Wouters (2003), for instance). The reconciliation of these apparently divergent findings has motivated a new area of research.

[1] Brinca et al. (2019*a*) has also analysed of the fiscal multiplier varies with the size of the shock

As suggested by Keane and Rogerson (2011), that branch of the literature has produced two different approaches for understanding the source of the puzzle. On the one hand, there is a set of studies that challenge the conventional mapping between the observations on labor supply fluctuations and the preference parameters implicitly assumed to have generated them. As most empirical applications hinge on those assumptions about the choice problem that generated the data, studies like Domeij and Floden (2006)[2] assessed how a different setting of the choice problem driving individual decisions on labor supply can account for the gap in elasticities. In particular, they show how ignoring liquidity constraints may bias downwards micro estimates when applying identifications methodologies as in Altonji (1986). Precisely, given that the Frisch elasticity governs the response in hours to an anticipated change in wage, it is closely related to the ability to smooth consumption. When faced with an expected wage increase, agents will intend to smooth it into consumption over time and this entails current dissaving that will be met by an increase in hours worked when the wage rise matrializes. Hence, to the extent that agents cannot anticipate consumption due to binding credit constraints, the incentive to intertemporally substitute labor supply is reduced. This, in turn, implies that assuming complete financial markets in devising one's identification strategy will take a less responsive labor supply due to binding borrowing constraints for a smaller Frisch parameter – and the one which would be captured in the absence of borrowing constraints.

On the other hand, other studies focused on whether a *de facto* moderately small Frisch parameter at the individual level may come to terms with a larger response at the aggregate level. In fact, since the earlier microeconometric studies have almost exclusively estimated the Frisch out of hours fluctuations on the intensive margin, findings as in Rogerson and Wallenius (2009), suggesting that adjustments at the extensive margin may explain the gap between the micro and macro evidence, motivated researchers to look into the micro data evidence on the intertemporal substitution of the labor supply on the extensive margin. However, most of this work – surveyed

[2] Similarly, Imai and Keane (2004) and Wallenius and Keane (2004) show that estimates will also be downward biased by failing to account for human capital accumulation. While Low (2005) also explores the potential for underestimation the Frisch parameter, in setting of uninsurable labor income risk where a precautionary savinggs motive renders hours worked less responsive to anticipated wage fluctuations.

in Chetty et al. (2011) – takes the elasticity of participation rates for the Frisch elasticity on the extensive margin, and, as argued by Peterman (2015), the two would only match if workers moving into employment, work, on average, the same as those already previously working. Peterman (2015) actually produces a micro data estimate of the Frisch driving hours fluctuations on both the extensive and intensive margin of 2.64, more in line with what the macro literature suggests.

Thus, the extensive literature leads us to say that, as much as with fiscal multipliers, there is no such think as *a* Frisch elasticity. Its value ultimately depends on the underlying choice problem and on the larger economic environment. In fact, if the extensive margin accounts for a big portion of the intertemporal substitution, then it may not only reflect preferences but also the technology of job search and of hours adjustments, for instance. Thus, in different countries, the Firsch elasticity underlying both extensive and intensive margin adjustments might be closer or further from the macro literature benchmark. This is precisely what motivates our stance regarding the magnitude of this parameter. Rather than adding to the on-going debate, we explore cross-country variation of Frisch that emerges from different magnitudes of both the extensive and intensive margin adjustments.

We therefore produce in section 2 country-specific estimates of the Frisch elasticity using the approach presented in Peterman (2015). These are then used to calibrate a macro model previously used to assess responses to fiscal consolidation programs (Brinca et al. (2019*b*)) and described in section 3. In section 4 we detail the calibration procedure and in section 5 we analyze the importance of country specific calibration of the Frisch elasticity to study the aggregate effect of fiscal stimulus. Section 6 concludes.

2 Frisch Elasticity

The identification approach of the seminal work on the Frisch estimation (Altonji (1986) and MaCurdy (1981)) was based on a simple life-cycle model of labor supply described in Equation 1. Recent studies that propose to solve the Frisch puzzle also used it as a benchmark, and it is the starting point of our analysis. At any given point in time, an individual decision on consumption,

c_t, and of hours worked, n_t, is such that:

$$\max_{c_t,n_t} \quad E_0 \sum_{t=0}^{T} \beta^t \Big(\frac{c_t^{1-\sigma}}{1-\sigma} - \chi_t \frac{n_t^{1+\frac{1}{\eta_t}}}{1+\frac{1}{\eta}} \Big) \tag{1}$$
$$\text{s.t.:} \quad c_t + a_{t+1} = w_t h_t + (1+r_t) a_t$$

The following first order conditions apply:

$$\begin{aligned} \lambda_t &= c_t^{-\sigma} \\ \lambda_t w_t &= \chi_t n_t^{1/\eta} \\ \lambda_t &= E_t \beta \Psi_{t,t+1} (1+r_t) \lambda_{t+1} \end{aligned}$$

where λ is the marginal utility of wealth. By taking the logs of each and further manipulating, one arrives at:

$$\Delta ln(n_t) = \eta \Delta ln(w_t) + \eta [\xi_{t+1} - ln\beta - ln(1+r_t) - \Delta ln(\chi_{t+1})] \tag{2}$$

where ξ_{t+1} is the unexpected change in marginal utility. As the Frisch elasticity is defined as the percentage change in hours due to a percentage change in wages, holding constant the marginal utility of wealth [3], the main challenge to identify η is to extract the variation of wages that is not associated with unanticipated changes - i.e., not correlated with surprises in the marginal utility, λ.

2.1 *Estimation strategy*

A natural approach for the estimation of η is to rely on the life-cycle profile of wages as the source of anticipated wage variation. To additionally clean this predictable wage fluctuations from correlation with a life-cycle trend in taste parameters of working, χ, one should also control for changes in tastes. This led Altonji (1986) and Peterman (2015) to consider the following estimation equation:

$$\Delta ln(n_t) = \eta \Delta ln(w_t) + \delta + \zeta \Delta TS_t + \varepsilon_t \tag{3}$$

[3] It can be shown that, given the specified utility function - homothetic and separable, in consumption and labor; $\eta = \frac{dn_t/n_t}{dw_t/w_t}|_\lambda$

where ΔTS_t is a vector of controls for taste parameters and δ a vector of year dummies to control for interest rate changes. By definition, Equation 3 can only extract the Frisch parameter underlying fluctuations of hours on the intensive margin, since it implies excluding non working individuals. Peterman (2015) thus proposed to use a pseudo panel approach [4]. It implies grouping individuals into cohorts – defined by year of birth, e.g., – and taking the average of each variable within the cohort, in every year. Equation 3 is then estimated with the panel of cohorts rather than the panel of individuals. In this way, the Frisch is estimated using information on both intensive and extensive margin adjustments. However, although this will make use of hours fluctuation of those moving in and out of employment, it does not use the variation of wages of these individuals, as these are unobserved. Since such is likely to exacerbate the point estimates we follow Peterman (2015) in his application of this strategy with the predicted wages for the individuals not working. In practice, this means predicting the wage of those not working and then proceed to produce the pseudo panel using these predicted wages[5][6].

2.2 Data

Performing labor market analysis across countries is often unfeasible due to lack of appropriate data that typically consists of a panel with information on both hours and wages. For EU countries, the Labor Force Survey (LFS) is the best reference for hours fluctuations - as it provides a long panel at a quarterly frequency - but lacks information on labor income. On the other hand, the EU Statistics on Living Conditions (EU-SILC) contains net labor income for several member states but with seemingly limit information on hours worked. To be precise, EU-SILC provides, at a yearly frequency, information on the weekly hours usually worked at the time of the interview and 12 variables on the employment status for each month. As described in Engel and Schaffner (2012), these can be combine to form a monthly version of the dataset with hours worked per month: from

[4] First suggested by Deaton (1985) to allow for panel data analysis when only cross-section data was available.

[5] Like in Peterman (2015) the wages of non-working individuals were estimated using a Heckman-type correction for selection bias. Specifically, we proceed as described in Wooldridge (2010)

[6] Is worth noting that handling a pseudo panel as a genuine panel is not without caveats. We rely on the results of Verbeek and Nijman (1992) that suggests the effects of taking a synthetic panel for a true panel will be attenuated with a large enough cohorts, of at least 100 observations.

which annual hours are easily computed [7].

Similar to Peterman (2015) and Altonji (1986) we calculate the hourly wage by dividing net labor income by annual hours worked and deflate them by the HICP. Further, observations with 250 per cent increase or a 60 percent decrease in wages were treated as missing, as well as observations with swings of more than 13. We define cohorts by the year of birth and when calculating the cohorts average we use the base weights adjustment proposed in Bors (2018).

2.3 *Results*

Table 1 reports our results for the Frisch estimation following Peterman (2015) approach[8]. This entails the use of instrumental variables to estimate η in Equation 3, which includes age polynomials, education dummies, interactions between the two and sex dummies [9]. The set of controls aims at controlling for changes in the disutility of work that are correlated with age. Year fixed effects are used for all countries.

It is striking that the Frisch seems to vary widely across countries. Some point estimates are closer to the micro evidence benchmark while others are more in line with the macro literature. For our set of countries, the Frisch spans from 0.7 to 4.36 with and average of 2.02 and a standard deviation of 1.2.

One possible limitation of this approach is the weakness of the IVs used. To cope with this, Equation 3 was estimated with the continuously-updated GMM (the CUE of Hansen)[10], which is known to be more robust to weak identification bias. Further, it is reassuring to see that there seems to be no correlation between a lower F-stat of excluded instruments and the magnitude of the point estimates. This suggests that the variation in the estimated Frisch is not driven by weak identification bias but rather true cross-country different in this deep parameter of labor supply.

[7] As this annual hours series is central in our analysis we compare it against the LFS benchmark in appendix A: the structural break for Sweden is indisputable, however, we obtain similar results when focusing on the last periods of the sample.

[8] The US results are taken from Peterman (2015) table 11.

[9] The set of IVs is not identical to all countries, please refer to Appendix B.

[10] This is the GMM generalization of the LIML (Limited Information Maximum Likelihood) estimator to the case of possibly heteroskedastic and autocorrelated disturbances.

Table 1. **Frisch Elasticity**

Variables (robust s.e.)	**CZ**	**EE**	**EL**	**ES**	**FR**	**LT**	**LV**	**PL**	**PT**	**SE**	**US**
Δ ln(w)	2.02	1.21	3.68	2.39	4.36	1.53	1.08	2.23	0.70	1.01	2.64
	(0.54)	(0.67)	(0.98)	(0.98)	(1.29)	(0.66)	(0.52)	(0.36)	(0.37)	(0.54)	(0.44)
Δ (Big city)	-0.8	-0.32	-0.17	0.15	-0.06	-0.50	-0.55	-0.84	0.02	0.13	0.41
	(0.21)	(0.28)	(0.28)	(0.29)	(0.36)	(0.33)	(0.19)	(0.25)	(0.22)	(0.19)	(0.29)
Δ (Medium-sized city)	-0.01		-0.38	0.54	-0.24	0.60		-0.21	0.36	0.34	
	(0.23)		(0.41)	(0.40)	(0.46)	(0.49)		(0.28)	(0.23)	(0.19)	
Δ (Kids under 6)		0.03	-0.01		-0.01	-0.05	0.23	-0.24	-0.20	0.01	-0.21
		(0.18)	(0.28)		(0.21)	(0.26)	(0.24)	(0.20)	(0.20)	(0.10)	(0.08)
Δ (Kids)			-0.20		-0.06	-0.32	-0.20	-0.05	-0.04		-0.21
			(0.18)		(0.12)	(0.19)	(0.14)	(0.10)	(0.11)		(0.07)
Year dummies	x	x	x	x	x	x	x	x	x	x	x
HH type dummies	x			x						x	
Observations	385	245	455	420	420	245	385	385	420	420	1,288
Ages	25-60	25-60	25-60	25-60	25-60	25-60	25-60	25-60	25-60	25-60	20-65
Years	05-16	09-16	03-16	04-16	04-16	09-16	05-16	05-16	04-16	04-16	68-97
1st Stage											
F-stat (Excl. Inst.)	13.51	3.55	3.87	2.76	4.58	2.48	4.96	27.29	4.76	2.12	8.22
F-stat (P-value)	0.00	0.01	0.01	0.02	0.00	0.05	0.00	0.00	0.00	0.02	0.00
J-Stat (P-value)	0.34	0.50	0.25	0.20	0.58	0.66	0.33	0.13	0.23	0.97	0.26
Stock and Yogo (2005) critical values											
Cragg-Donald Wald F-stat	11.47	2.20	3.93	2.77	3.61	2.99	5.15	20.64	4.76	2.24	
10% maximal size	5.44	4.45	5.44	4.84	5.44	4.45	4.45	4.84	4.84	3.21	
15% maximal size	3.87	3.34	3.87	3.56	3.87	3.34	3.34	3.56	3.56	2.34	
20% maximal size	3.3	2.87	3.3	3.05	3.3	2.87	2.87	3.05	3.05	2.06	
25% maximal size	2.98	2.61	2.98	2.77	2.98	2.61	2.61	2.77	2.77	1.9	

3 Model

This section describes the model used to study the role of the Frisch elasticity on the impact of fiscal consolidation programs in different economies. It is a standard neoclassical model with overlapping generations but with incomplete markets as in Brinca et al. (2019*b*), which proposed the inclusion of a bequest motive for retirees to better capture dynamics of wealth accumulation over the life cycle.

3.1 Technology

The representative firm uses capital , K_t, and labor efficiency units, L_t, in the productive process according to Cobb-Douglas production function:

$$Y_t(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha} \tag{4}$$

The capital stock law of motion is defined by:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{5}$$

where δ is the depreciation rate of capital in each period and where I_t is gross investment in period t. The decision on input units is driven, in each period, by profit maximization:

$$\max_{L_t, K_t} \Pi_t = Y_t - [w_t L_t + (r_t + \delta)K_t] \tag{6}$$

which in a perfectly competitive setting with flexible prices implies that each factor price equates its marginal product:

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} \tag{7}$$

$$r_t = \frac{\partial Y_t}{\partial K_t} = \alpha \left(\frac{L_t}{K_t}\right)^{1-\alpha} - \delta \tag{8}$$

3.2 Labor income

At any given period, each households' endowment of efficiency units of labor is determined by its permanent ability a - where $a \sim N(0, \sigma_a)$ -, its age, j, and a household-specific idiosyncratic productivity shock, u_t, characterized by an AR(1) process:

$$u_{t+1} = \rho u_t + \varepsilon_{t+1}, \varepsilon \sim N(0, \sigma_\varepsilon^2) \tag{9}$$

It follows that an individual's wage is given by:

$$w(j, a, u) = w e^{\gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + a + u} \tag{10}$$

where γ_1, γ_2 and γ_3 accounts for the age profile of labor efficiency units, and where w is the wage per each of those units, determined in a competitive labor market.

3.3 Demographics

There are J overlapping generations in our economy. Each new household has age 20 and every household retires at 65. A model period corresponds to one year, meaning that active work-live spans over 40 model periods. Let j denote households' age. Those retired face a probability of dying $\pi(j)$, dependent on age; and no household lives longer than 100[11]. We assume population size is fixed. By normalizing each new cohort's size to one, according to the law of large numbers, the mass of retired households is given by $\Omega_j = \prod_{q=65}^{q=J-1} \omega(q)$, where $\omega(j) = 1 - \pi(j)$ denotes the survival probability.

Households are heterogeneous across asset holdings, idiosyncratic productivity, permanent ability, their subjective time discount factor and, of course, age. The discount factor is time invariant and is one of three values , $\beta \in \{\beta_1, \beta_2, \beta_3\}$, uniformly distributed across each generation.

[11] Implying that J=81

Permanent ability is realized at birth. During their work-life, agents decide how many hours to work, n, and how much to consume, c, in each period. When retired, they make no decision on labor supply and receive the social security payment, Ψ_t.

Given there are no annuity markets, the bequests left by deceased households are equally redistributed among all other households through a lump-sum transfer denoted by Γ.

3.4 Government

The government's behaviour in the model is characterized as follows. The social security system is always balanced, with the proceeds from taxing employees and employers - at rates τ_{ss} and $\tilde{\tau}_{ss}$ - exactly offsetting the benefits, Φ_t, paid to retirees. Taxes on consumption, labor and capital income are used to finance government expenditures on pure public goods, G_t [12], lump-sum transfers, g_t, and interest on public debt, $r_t B_t$. It is assumed that there is some outstanding public debt and that this evolves so as to keep the debt-to-output ratio, $B_t = B_t/Y_t$, constant over time[13]. While consumption and capital income are taxed at flat rates, labor income is taxed in a non-linear fashion. We use the functional form proposed in Benabou (2002) and argued to fit the US data appropriately in Heathcore, Storesletten and Violante (2017):

$$\tau(y) = 1 - \theta_0 y^{-\theta_1} \tag{11}$$

where $\tau(y)$ stands for the average tax rate applied to pre-tax labor income of y. Parameters θ_0 and θ_1 capture the level and progressivity of the tax schedule, respectively[14]. Finally, if one takes R_t for total tax revenue and R_t^{ss} for total the total revenue form social-security contributions, the government faces in the steady state the following budget constraint:

$$g\left(45 + \sum_{j \geq 65} \Omega_j\right) = R - G - rB \tag{12}$$

[12] These enter separably in the utility function

[13] Such implies the ratio of government revenues-to-output and government expenditures-to-output are constant in the steady state.

[14] A further discussion of the properties of this tax function is provided in the appendix C

$$\psi\left(\sum_{j\geq 65}\Omega_j\right) = R^{SS} \tag{13}$$

3.5 Preferences

A household's decision is driven by a monetary utility function, $U(c,n)$, dependent on consumption and hours worked, $n \in [0,1[$, defined as:

$$U(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \chi\frac{n^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \tag{14}$$

with σ representing risk aversion, χ the disutility of working and η the Frisch elasticity. In this model wealth accumulation over the life cycle is also influenced by a bequest motive, since retirees obtain utility from the wealth stock they leave to the other generations when they die:

$$D(k) = \phi log(k) \tag{15}$$

3.6 Recursive formulation of household problem

Any household is characterized, at each point in time, by the vector (k,β,a,u,j). Where k denotes household's savings, β the time discount factor, a permanent ability, u the idiosyncratic productivity shock, and j the household's age. Thus, for households of working-age, the decision problem on hours worked, n, consumption c, and future net assets holdings,k', can be formulated as:

$$\begin{aligned}
V(k,\beta,a,u,j) &= \max_{c,k',n}\left[U\left(c,n\right)+\beta E_{u'}\left[V(k',\beta,a,u,j+1)\right]\right] \\
&s.t.: \\
c(1+\tau_c)+k' &= (k+\Gamma)\left(1+r(1-\tau_k)\right)+g+Y^L \\
Y^L &= \frac{nw\left(j,a,u\right)}{1+\tilde{\tau}_{SS}}\left(1-\tau_{SS}-\tau_l\left(\frac{nw\left(j,a,u\right)}{1+\tilde{\tau}_{SS}}\right)\right) \\
n \in]0,1], &\quad k' \geq -b, \quad c>0
\end{aligned} \tag{16}$$

Y^L denotes labor income net of social contribution taxes and labor income taxes. $\tau_s s$ and $\tilde{\tau}_{ss}$ denote the rates of the social-security contributions paid by employee and employer, respectively. As for a retired household facing a probability $\pi(j)$ of dying and who obtains utility form any bequest left, the decision problem is defined by:

$$
\begin{aligned}
V(k,\beta,j) &= \max_{c,k'} \left[U(c,n) + \beta(1-\pi(j))V(k',\beta,j+1) + \pi(j)D(k) \right] \\
& s.t.: \\
c(1+\tau_c) + k' &= (k+\Gamma)(1+r(1-\tau_k)) + g + \psi \\
k' &\geq 0, \quad c > 0
\end{aligned}
\tag{17}
$$

3.7 Stationary recursive formulation of competitive equilibrium

With $\Phi(k,\beta,a,u,j)$ defining the measure of households with the corresponding characteristics, the stationary recursive equilibrium is defined by:

1. Given the factor prices and the initial conditions, the value function $V(k,\beta,a,u,j)$ and the policy functions, $c(k,\beta,a,u,j)$, $k'(k,\beta,a,u,j)$, and $n(k,\beta,a,u,j)$, solve the household optimization problem.

2. The factor prices are such that:
$$
w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}
$$
$$
r = \alpha\left(\frac{L}{K}\right)^{1-\alpha} - \delta
$$

3. The government budget balances:

$$
g\int d\Phi + G + rB = \int \left(\tau_k r(k+\Gamma) + \tau_c c + n\tau_l \left(\frac{nw(a,u,j)}{1+\tilde{\tau}_{SS}} \right) \right) d\Phi
$$

4. The social-security system balances:

$$\psi \int_{j \geq 65} d\Phi = \frac{\tilde{\tau}_{SS} + \tau_{SS}}{1 + \tilde{\tau}_{SS}} \left(\int_{j \geq 65} nwd\Phi \right)$$

5. The assets of the deceased are uniformly distributed among the living:

$$\Gamma \int \omega(j) d\Phi = \int (1 - \omega(j)) k d\Phi$$

3.8 *Fiscal experiment and transition*

The fiscal experiment considered is identical to that of Brinca et al. (2019*b*), and consists of an unanticipated fiscal consolidation program that reduces the debt-to-GDP ratio in 10 percentage points over 50 years. Concretely, we focus on the case of a gradual decrease of government purchases, G, of 0.2% of the steady-state GDP per year. At the end of the consolidation program, public spending returns to its initial level, and public transfers adjust in order to balance the public budget. It is assumed that the economy takes another 50 years to converge to the new steady state with the reduced debt-to-GDP ratio.

The appendix D defines the transition equilibrium subsequent to the fiscal experiment. The main modification is that the dynamic programming problem of households requires a new state variable: time, t, that controls for all the changes in policy and price variables relevant for the their optimization. Like in Brinca et al. (2016) and Krusell and Smith (1999), the numerical solution of the model needs guessing on paths for all variables dependent on time and solving this maximization problem backwards, subsequently updating the guess.

3.9 *Definition of the fiscal multiplier*

In the context of either fiscal consolidation policy, we distinguish between the impact multiplier and the cumulative multiplier. For the debt reduction through cuts in government spending, G, the impact multiplier is given by:

$$impact\ multiplier\ G = \frac{\Delta Y_0}{\Delta G_0} \tag{18}$$

with ΔY_0 being the output change from period 0 to period 1 and ΔG_0 the analogous for government spending. The cumulative multiplier is then given by:

$$cumulative\ multiplier\ G(T) = \frac{\sum_{t=0}^{t=T} \left(\prod_{s=0}^{s=T} \frac{1}{1+r_s} \right) \Delta Y_t}{\sum_{t=0}^{t=T} \left(\prod_{s=0}^{s=T} \frac{1}{1+r_s} \right) \Delta G_t} \tag{19}$$

with ΔY_t being the output change from period 0 to period t and ΔG_t the analogous for government spending.

4 Calibration

The model described in the preceding section was calibrated for 6 different countries: France, Spain, Portugal, Greece and the United States. Whilst some of the model parameters have straightforward empirical counterparts - and can therefore be calibrated exogenously -, others represent theoretical concepts not directly observed in the data. Hence, the latter are calibrated by means of a Simulated Method of Moments (SMM) approach, whereby these endogenously calibrated parameters are set such that the model replicates some targeted data moments of each economy.

4.1 Wages

For the parameters governing the live-cycle profile of wages we resort to the procedure adopted in Brinca et al. (2016) and Brinca et al. (2019*b*). The authors estimate, for each country, the empirical counterpart of equation 10, using data from the Luxembourg Income Survey (LIS):

$$ln(w_i) = ln(w) + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + \varepsilon_i \tag{20}$$

with j as the age of individual i. Regarding the persistence of the idiosyncratic shock, ρ, their estimate results from PSID data for the US and which we apply to all countries. Likewise, for the European countries, we take the average of the calibrated values of σ_a, the variance of time-invariant ability, used in Brinca et al. (2016) and the corresponding value for the US. The variance of the idiosyncratic shock, σ_ε, is then fixed to replicate the variance of the log of wages in each country.

4.2 *Preferences and borrowing limit*

The set of parameters calibrated endogenously also includes the disutility from work, χ, the bequest motive, φ, the discount factors, β_1, β_2 and β_3, and the borrowing limit, b. These are chosen in order to replicate, respectively, the observed fraction of hours worked per year, the ratio between the average of wealth held by households from 75 to 80 years old and overall average wealth [15], the capital ratio K/Y, and the three wealth quartiles. As for the Frisch elasticity, we take the point estimates from table 1.

4.3 *Taxes and social security*

The rates of the social-security contribution of employee and employer are assumed to be flat. We take the respective average between 2001 and 2007 from OECD. The consumption and capital income tax rate, are also assumed flat and are calculated following Trabandt and Uhlig (2011). As for the labor income tax schedule described in equation 11, we follow Brinca et al. (2019*b*) by first, estimating θ_0 and θ_1 for several types of households using OECD data on labor income tax, and then, taking the weighted average of θ_0 and θ_1 [16], so as to apply to all households in the model [17].

[15] As in Brinca et al. (2019*b*) we take the figure for the US economy as the benchmark for all countries.
[16] The weights used are the share of each family in the total population in the U.S. data
[17] Appendix E summarizes these figures for all countries calibrated

4.4 *Parameters calibrated endogenously*

The SMM approach is used to calibrate the parameters with no direct empirical counterpart: φ, β_1, β_2, β_3, b, χ and σ_e. It consists of minimizing the loss function:

$$L(\varphi,\beta_1,\beta_2,\beta_3,b,\chi,\sigma_\varepsilon) = ||M_m - M_d|| \tag{21}$$

where M_m and M_d are the data moments for the model and the data, respectively. As described previously, we have the same number of parameters to be endogenously calibrated and targeted moments. This ensures the system is exactly identified. Table 2 summarizes the the calibration fit for the U.S., please refer to table 5 for a more detailed summary for all countries. In the case of the U.S., we achieve an average error margin of 0.678%.

Table 2. **U.S. calibration fit**

Data Moment	Description	Source	Data Value	Model Value
$\tilde{a}_{75}/\bar{a}$	Mean wealth age 75-80 / mean wealth	LWS	1.513	1.511
K/Y	Capital-output ratio	PWT	3.073	3.078
Var(ln$w$)	Variance of log wages	LIS	0.509	0.509
$\tilde{n}$	Fraction of hours worked	OECD	0.248	0.248
Q_{25}, Q_{50} Q_{75}	Wealth Quartiles	LWS	-0.014,0.004, 0.120	-0.013,0.000,0.124
$L(\varphi,\beta_1,\beta_2,\beta_3,b,\chi,\sigma_\varepsilon)$				0.678

5 Results

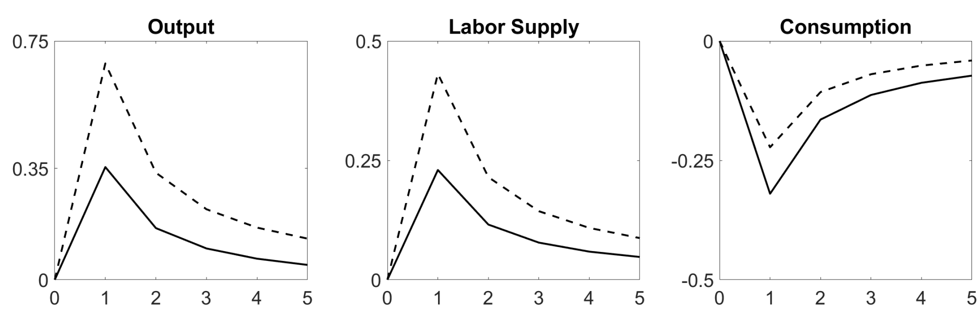
5.1 *Inspecting the mechanisms*

In the fiscal experiment we consider the government conducts a debt consolidation program that aims to reduce debt-to-output ratio by 10 percentage points over 50 periods. This is achieved by a reduction in public spending of 0.2% of the steady-state output, in each of those 50 periods, that is unanticipated. Our main purpose is then to understand how the effects of this policy are shaped by the country-specific structural behaviour of the labor supply.

In our neoclassical macro model, the responsiveness of labor supply is crucial to define the transition path to the new steady-sate of a lower public debt ratio, starting from the one previously calibrated. The decrease in public expenditure, G, triggers a crowding-in effect on private investment that spurs the accumulation of productive capital in the following years. Then, as the capital per worker ratio increases the productivity of each labor efficiency unit, real wages, will also rise in the future. In turn, with households perceiving the increase in their life time income, there is an immediate increase in private consumption and also a reallocation of labor supply over time. An intertemporal substitution effect that lures households to exchange a more expensive future leisure for more leisure today drives them to reduce hours supplied in the immediate short-run, and thus leading to a decrease in output.

It is precisely the Frisch that governs the magnitude of such substitution effect. A higher Frisch means a larger responsiveness to an expected rise in future wages and entails a greater intertemporal substitution effect. However, as pointed out by Keane and Rogerson (2011), the link between the Frisch and the intertemporal substitution will be muted by incomplete markets. Since credit constrained agents will be unable to borrow against their higher future income, they face less incentives to concentrate their life-time labor supply in the periods of higher wages, and consequently attenuate aggregate response today.[18].

Figure 1: **Cumulative multipliers over the first 5 periods**



Output cumulative multiplier (left panel), Labor Supply cumulative multiplier (middle panel) and Consumption cumulative multiplier (right panel) in the first 5 periods in Spain (dashed line) and Portugal (solid line) .

Figure 1 illustrates this mechanisms by comparing the response to the fiscal consolidation of Spain and Portugal. These are specially suited for comparison given the identical fraction of credit constrained households in the two economies. The gap in both countries' response in terms of labor

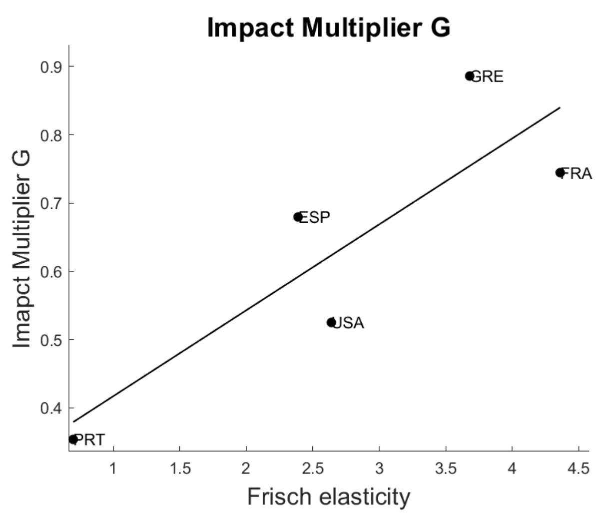
[18] Bernardino (2019) finds this effect to be specially relevant when accounting for differences in the distribution of liquid wealth.

supply is thus imputable to a Frisch elasticity in Spain of 2.39 with respect to 0.7 for Portugal.

5.2 *Cross-country analysis*

In order to quantitatively validate the role of Frisch elasticity on the fiscal multiplier we expand our analysis to the entire set of calibrated countries. The model allows for a reasonable set of country characteristics to shape its aggregate response. These include the wealth distribution, and the closely related fraction of liquidity constrained households, the degree of labor income risk that entails a precautionary savings motive and different time-discounting preferences. Although all of these come into play at varying degrees, Figure 2 emphasizes the relevance of accounting for cross-country differences in the Frisch. It depicts a strong quantitative and positive relationship between the elasticity and the multiplier of the fiscal experiment that is statistically significant. Further, it also corroborates the described interplay between the Frisch and the share of credit constrained agents in the economy. With a comparable Frisch elasticity the gap in the response of Spain and the USA, and of Greece and France is imputable to a fraction of liquidity constrained agents that attenuates the sensitivity of the labor supply adjustment [19].

Figure 2: **The Frisch and the impact multiplier**



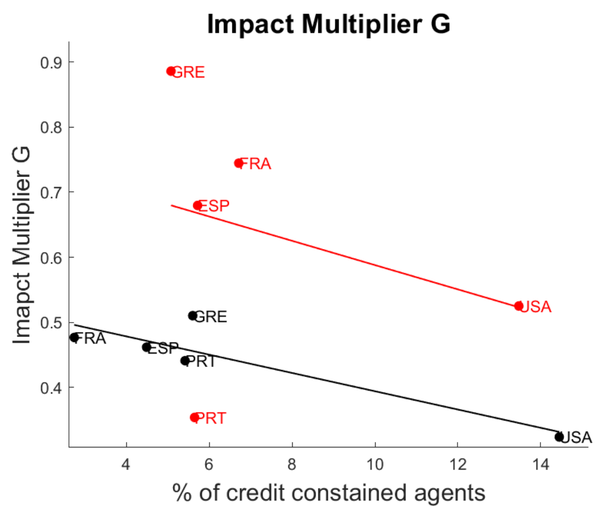
The impact multiplier and the estimated Frisch elasticity (table 1). The correlation coefficient is 0.858 with a p-value of 0.063

Finally, we assess the added explanatory power of a calibration with country specific Frisch as

[19] The USA calibration produces 13.47% of credit constrained households with respect to 5.72% for Spain. For France this Figure is 6.71% in comparison to 5.08% for Greece

oppose to the standard calibration procedure of setting this elasticity equal to one. The red dots in Figure 3 depict our calibration of a country-varying Frisch while the black dots depict the standard calibration. With the former procedure, our model is able to produce a much wider variation of impact multipliers, ranging from 0.35 to 0.9. Yet again, the Figure shows that such calibration enables the model to replicate a statistically significant interplay between credit constraints and the deep parameter governing much of the intertemporal substitution of labor supply. The country-specific Frisch calibration mutes, quantitatively and statistically, the relationship between liquidity constraints and fiscal multipliers.

Figure 3: **Credit constrained agents and the impact multiplier**



The impact multiplier and the fraction of credit constrained agents replicated by the model. In red the model calibration allowing for a country-specific Frisch (correlation coefficient is -0.317 with a p-value of 0.603), in black the model calibration where the Frisch is set to 1 for all countries (correlation coefficient is -0.9016 with a p-value of 0.037).

6 Conclusion

This work contributes to the literature explaining variation on fiscal multipliers through a supply-side perspective. By modelling an overlapping generations economy from a neoclassical standpoint, with uninsurable labor income risk and liquidity constraints, we produce evidence suggesting that the variation of the Frisch elasticity parameter across countries may explain a large dispersion of fiscal multipliers. Although the link between the Frisch parameter and the actual aggregate intertemporal substitution response of the labor supply is deteriorated by the fraction of liquidity constrained agents or the existence of uninsurable labor income risk, our supply-side driven economy setting produces a quantitatively relevant positive relationship between the Frisch parameter and the size of the fiscal multiplier.

Micro data evidence suggesting substantially country heterogeneity on the Frisch parameter underlying hours fluctuations on both the extensive and intensive margin, emphasizes the importance of this mechanism for further analysis on fiscal policy. Based on our results an interesting future research avenue would be to deepen the analysis of the interplay between the rivaling labor supply driven mechanisms in shaping the response to fiscal stimulus. Our set of 5 modeled countries had limited variation on the fraction credit constrained agents and recent work by Bernardino (2019) showed that accounting for the liquid wealth distribution may reveal a stronger effect of liquidity constraints on the labor supply.

References

Altonji, Joseph G. 1986. "Intertemporal Substitution in Labor Supply: Evidence from Micro Data." *Journal of Political Economy*, 94(3): S176–S215.

Benabou, R. 2002. "Tax and education policy in a heterogeneous agent economy: What levels of redistribution maximize growth and efficiency." *Econometrica*, 70: 481–517.

Bernardino, Tiago. 2019. "Asset Liquidity and Fiscal Consolidation Programs." MPRA Paper 93903, University Library of Munich.

Bors, Marwin. 2018. "First computational steps towards a cumulative sample based on the EU-SILC longitudinal datasets." GESIS Papers No. 11.

Brinca, Pedro, A. Hans Holter, Per Krusell, and Laurence Malafry. 2016. "Fiscal multipliers in the 21st century." *Journal of Monetary Economics*, 77: 53–69.

Brinca, Pedro, Miguel Faria-e Castro, Miguel Homem Ferreira, and Hans Holter. 2019*a*. "The nonlinear effects of fiscal policy." *FRB St. Louis Working Paper*, , (2019-15).

Brinca, Pedro, Miguel H. Ferreira, Francesco Franco, A. Hans Holter, and Laurence Malafry. 2019*b*. "Fiscal Consolidation Programs and Income Inequality." MPRA Paper 82705, University Library of Munich.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins." *American Economic Review*, 101(3): 471–475.

Deaton, A. 1985. "Panel Data from Time Series of Cross-Sections." *Journal of Econometrics*, 30: 109–126.

Domeij, David, and Martin Floden. 2006. "The labor-supply elasticity and borrowing constraints: Why estimates are biased." *Review of Economics Dynamics*, 9(2): 242–262.

Engel, Melissa, and Sandra M Schaffner. 2012. "How to Use the EU-SILC Panle to Analyse Monthly and Hourly Wages." RUHR Economic Papers No. 390.

Heathcore, J., S. Storesletten, and G. Violante. 2017. "Optimal tax progressivity: An analytical framework." *Quarterly Journal of Economics*, 132: 1693–1754.

Hotler, H., D. Krueger, and S. Stepanchuk. 2017. "How do tax progressivity and household heterogenity affect laffer curves?" Workin paper.

Imai, Susumu, and Michael Keane. 2004. "Intertemporal Labor Supply and Human Capital Accumulation." *International Economic Review*, 9(2): 242–262.

Keane, Michael P., and Richard Rogerson. 2011. "Reconciling Micro and Macro Labor Supply Elasticities: A Structural Perspective." NBER Working Paper No. 17430.

King, Robert G, and Sergio T Rebelo. 1999. "Resuscitating real business cycles." In *Handbook of Macroeconomics*. , ed. J.B. Taylor and M. Woodford, Chapter 14, 927–1007. Elsevier.

Krusell, P., and A. Smith. 1999. "On the welfare effects of eliminating business cycles." *Review of Economic Dynamics*, 2(1): 245–272.

Low, H. 2005. "Self-insurance in a life-cycle model of labour supply and savings." *Review of Economic Dynamics*, 8(4): 945–975.

MaCurdy, Thomas E. 1981. "An Empirical Model of Labor Supply in a Life-Cycle Setting." *Journal of Political Economy*, 89(6): 1059–1085.

Peterman, William B. 2015. "Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity." *Economic Inquiry*, 54: 100–120.

Reichling, F., and C. Whalen. 2012. "Review of estimates of the Frisch Elasticity of labor supply." Washington, DC: Congressional Budget Office.

Rogerson, Richard, and Johanna Wallenius. 2009. "Micro and macro elasticities in a life cycle model with taxes." *Journal of Economic Theory*, 144(6): 2277–2292.

Smets, Frank, and Rafael Wouters. 2003. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.

Stock, James, and Motohiro Yogo. 2005. "Asymptotic distributions of instrumental variables statistics with many instruments." *Identification and inference for econometric models: Essays in honor of Thomas Rothenberg*, 109–120.

Trabandt, M., and H. Uhlig. 2011. "The laffer curve revisited." *Journal of Monetary Economics*, 58(4): 305–327.

Verbeek, M., and T. Nijman. 1992. "Can Cohort Data be Treated as Genuine Panel Data." *Empirical Economics*, 17: 9–23.

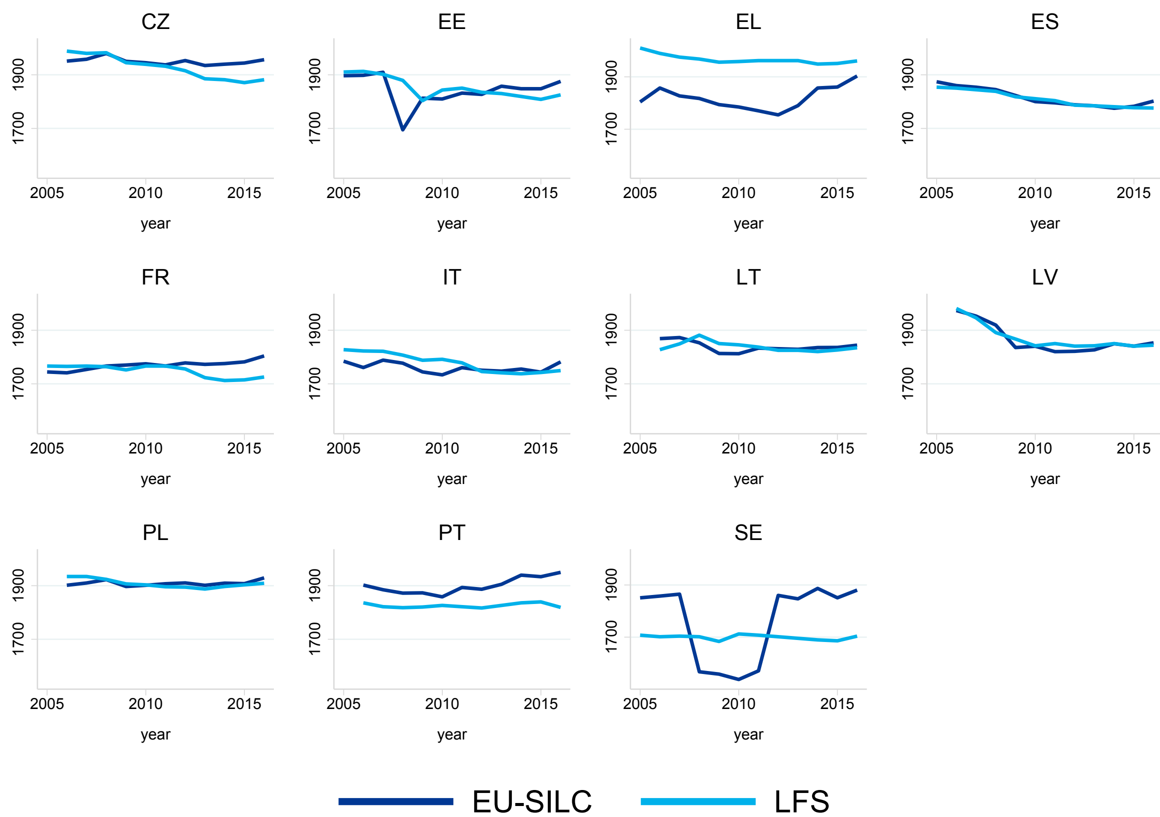
Wallenius, Johanna, and Michael Keane. 2004. "Human Capital Accumulation and the Intertemporal Elasticity of Substitution of Labor: How Large is the Bias?" *Review of Economic Dynamics*, 14(4): 577–592.

Wooldridge, J. 2010. *Econometric Analysis of Cross Section and Panel Data.* MIT Press.

Appendix

A Match of yearly hours with LFS

Figure 4: **Average yearly working-hours**



B Instrumental variables

Table 3. **Excluded instrumental variables**

IVs	CZ	EE	EL	ES	FR	LT	LV	PL	PT	SE
Age polynomial	x	x	x	x	x	x	x	x	x	x
Primary education		x		x	x	x	x		x	x
Tertiary education		x		x	x	x	x		x	x
Sex	x		x	x				x	x	
Occupation dummies										x
Occupation dummies x Age										x
Tertiary education										
Primary educ. x Age		x				x	x			
Tertiary educ. x Age		x				x	x			
Sex x Age	x		x					x		

C Tax Function

[20] Given the tax function

$$ya = \theta_0 y^{1-\theta_1}$$

which we employ, the average tax rate is defined as

$$ya = [1 - \tau(y)]y$$

and thus

$$\theta_0 y^{1-\theta_1} = [1 - \tau(y)]y$$

and thus

$$1 - \tau(y) = \theta_0 y^{-\theta_1}$$

[20] This appendix is borrowed from Hotler, Krueger and Stepanchuk (2017)

$$\tau(y) = 1 - \theta_0 y^{-\theta_1}$$

$$T(y) = \tau(y).y = y - \theta_0 y^{1-\theta_1}$$

$$T'(y) = 1 - (1-\theta_1)\theta_0 y^{-\theta_1}$$

Thus the tax wedge for any two incomes (y_1, y_2) is given by:

$$1 - \frac{1-\tau(y_2)}{1-\tau(y_1)} = 1 - \left(\frac{y_2}{y_1}\right)^{-\theta_1} \tag{22}$$

and therefore independently of the scaling parameter θ_0. Thus by construction one can raise average taxes by lowering θ_0 and not change the progressivity of the tax code, since (as long as tax progressivity is defined by the tax wedges) the progressivity of the tax code[21] is uniquely determined by the paramenter θ_1

D Definition of a Transition Equilibrium after the Unanticipated Fiscal Consolidation Shock

[22]We define a recursive competitive equilibrium along the transition between steady states as follows:

Given the initial capital stock, the initial distribution of households and initial taxes, respectively K_0, ϕ_0 and $\{\tau_l, \tau_c, \tau_k, \tau_{SS}, \tilde{\tau}_{SS}\}_{t=1}^{t=\infty}$, a competitive equilibrium is a sequence of individual functions for the household, $\{V_t, c_t, k_t', n_t\}_{t=1}^{t=\infty}$, of production plans for the firm, $\{K_t, L_t\}_{t=1}^{t=\infty}$, factor prices, $\{r_t, w_t\}$, government transfer $\{g_t, \Psi_t, G_t\}_{t=1}^{t=\infty}$, government debt, $\{B_t\}_{t=1}^{t=\infty}$, inheritance from the dead, $\{\Gamma_t\}_{t=1}^{t=\infty}$, and of measures, $\{\Phi_t\}_{t=1}^{t=\infty}$, such that for all t:

---

[21] Note that

$$1 - \tau(y) = \frac{1-T'(y)}{1-\theta_1} > 1 - T'(y)$$

and thus as long as $\theta_1 \in]0,1[$ we have that

$$T'(y) > \tau(y)$$

and thus marginal tax rates are higher than average tax rates for all incomes.

[22] This appendix is borrowed from Brinca et al. (2019*b*)

1. Given the factor prices and the initial conditions the consumers' optimization problem is solved by the value function $V(k,\beta,a,u,j)$ and the policy functions $c(k,\beta,a,u,j)$, $k'(k,\beta,a,u,j)$, and $n(k,\beta,a,u,j)$

2. Markets clear:

$$K_{t+1}+B_t=\int k_t\,d\Phi_t$$

$$L_t=\int n_t(k_t,\beta,a,u,j)\,d\Phi_t$$

$$\int c_t\,d\Phi_t+K_{t+1}+G_t=(1-\delta)K_t+K^{\alpha}L^{1-\alpha}$$

3. The factor prices satisfy:

$$w_t=(1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}$$

$$r_t=\alpha\left(\frac{L_t}{K_t}\right)^{1-\alpha}-\delta$$

4. The Government budget balances:

$$g_t\int d\Phi_t+G_t+r_tB_t=\int\left(\tau_k r_t(k_t+\Gamma_t)+\tau_c c_t+n_t\tau_l\left(\frac{n_t w_t(a,u,j)}{1+\tilde{\tau}_{SS}}\right)\right)d\Phi_t+(B_{t+1}-B_t)$$

5. The social security system balances:

$$\psi_t\int_{j\geq 65}d\Phi_t=\frac{\tilde{\tau}_{SS}+\tau_{SS}}{1+\tilde{\tau}_{SS}}\left(\int_{j\geq 65}n_t w_t d\Phi_t\right)$$

6. The assets of the dead are uniformly distributed among the living:

$$\Gamma_t\int\omega(j)_t d\Phi_t=\int(1-\omega(j))k_t d\Phi_t$$

7. Aggregate law of motion:

$$\phi_{t+1}=\gamma_t(\phi_t)$$

E Calibration: additional figures

Table 4. **Country-specific calibration targets and exogenous parameters**

	Macro ratios		Labor targets		Age profile parameters			Taxes					
	K/Y	B/Y	$\tilde{n}$	Var(ln$w$)	γ_1	γ_2	γ_3	θ_0	θ_1	$\tilde{\tau}_{ss}$	τ_{ss}	τ_c	τ_k
France	3.392	0.559	0.184	0.478	0.384	-0.008	6.0e-05	0.915	0.142	0.434	0.135	0.183	0.355
Greece	3.262	1.038	0.230	0.220	0.120	-0.002	1.3e-05	1.062	0.201	0.280	0.160	0.154	0.160
Portugal	3.229	0.557	0.249	0.298	0.172	-0.004	2.6e-05	0.937	0.136	0.238	0.110	0.208	0.234
Spain	3.378	0.368	0.204	0.250	0.114	-0.002	1.4e-05	0.904	0.148	0.305	0.064	0.144	0.296
USA	3.074	0.428	0.248	0.509	0.265	-0.005	3.6e-05	0.888	0.137	0.078	0.077	0.364	0.047

Macro ratios: K/Y is dreived from the Penn World Table 8.0, average from 1990-2011; B/Y is the average of net public debt from 2001-2008, IMF data.

Labor targets: $\tilde{n}$ comes from OECD data, year average from 1990-2011 ;Var(ln$w$) and $\gamma_1, \gamma_2 \gamma_3$ are estimated according to equation (20), using the most recent LIS survey available before 2008. Data for Portugal comes from Quadros de Pessoal 2009 database.

Taxes: θ_0, θ_1 are estimated as described in section 4.3; $\tilde{\tau}_{SS}, \tau_{SS}$ are the average social security taxes paid by the employer and by the employee, respectively, using OECD data of 2001-2007; τ_c and τ_k come from Trabandt and Uhlig (2011) or calculated using their approach and correspond to the average effective tax rate from 1995-2007.

Table 5. **Endogenous parameters and calibration fit**

Country	β_1	β_2	β_3	b	χ	φ	σ_ε	L
France	0.933	1.006	0.979	0.06	4.3	4.38	0.506	0.800
Greece	0.979	0.994	0.9775	0.00	5.5	4.3	0.121	1.630
Portugal	0.964	0.992	0.952	0.00	20.5	5.1	0.38	1.841
Spain	0.933	0.993	0.974	-0.05	9.2	5.6	0.237	2.704
USA	0.988	0.924	0.915	0.175	5.0	6.24	0.307	0.678

The last column displays the value of the loss function defined in equation 21. It can be interpreted as the average error margin (%).